

Commutative Algebra
(MMath-II, End-Sem. Exam-2020)

Instructions :- 1. Duration 6 hours, additional 30 minutes to upload the answer-script on Moodle.

2. You may quote and use any (and only) result covered in the course, without proof.

3. Solve problems for a maximum score of 40.

→ Contacts: maneesh.thakur@gmail.com, 9650330176

1. Find polynomial subalgebras R and S of the K -algebra $A = K[X, Y] / \langle X^2 - Y^3 \rangle$, K a field, X, Y are variables over K , such that A is integral over both R and S . What is $\text{Krull dim}(A)$? Explain. [10]
2. Let A be an affine K -algebra, K -a field, with $\text{Spec}(A)$ finite. Prove that the K -vector space A is finite dimensional. Is A Artinian? Explain. [10]
3. Let K be a field. Prove that any K -subalgebra of the polynomial algebra $K[x]$ is finitely generated. Is this true for $K[X, Y]$? Explain. [10]
4. Let A be a Dedekind domain and M a flat A -module. Is M necessarily projective over A ? Explain. [10]
5. Let K be a field; x, y variables over K . Compute the Krull dimension of $K[X, Y] / \langle X^2 - XY \rangle$. [10]